

34.11 A heat recovery system repurposes waste heat from an industrial process in the form of hot flue gas ($c_p = 0.25 \frac{Btu}{lb^\circ F}$). $2 \times 10^6 \frac{lb_m}{hr}$ of $1000^\circ F$ flue gas is available from the process. Prior to entering a gas turbine, the flue gas undergoes supplemental heating with $100,000 \frac{lb_m}{hr}$ of natural gas initially at $70^\circ F$ being combusted at constant pressure. Natural gas has a higher heating value of $17,000 \frac{Btu}{lb_m}$ and a specific heat capacity of $0.7 \frac{Btu}{lb^\circ F}$. What is the final temperature of the flue gas after combustion?

- A. $4270^\circ F$
- B. $4480^\circ F$
- C. $4730^\circ F$
- D. $4940^\circ F$

Start by calculating how much heat is added in the combustor by burning the natural gas. The heat added is the product of the mass flow rate of fuel and the heating value.

$$\dot{Q}_{combustion} = \dot{m}_{fuel} (HHV)$$

$$\dot{Q}_{combustion} = \left(100,000 \frac{lb_m}{hr} \right) \left(17,000 \frac{Btu}{lb_m} \right) = 1.7 \times 10^9 \frac{Btu}{hr}$$

Most of this heat will go toward heating the flue gas, however some will be used up in raising the temperature of the fuel itself from its initial temperature up to the starting temperature of the flue gas. Calculate this quantity and subtract from the heat of combustion to determine how much heat is added to the flue gas.

$$\dot{Q}_{htg\ fuel} = \dot{m}_{fuel} c_p \Delta T$$

$$\dot{Q}_{htg\ fuel} = \left(100,000 \frac{lb_m}{hr} \right) \left(0.7 \frac{Btu}{lb^\circ F} \right) (1000^\circ F - 70^\circ F) = 6.51 \times 10^7 \frac{Btu}{hr}$$

$$\dot{Q}_{flue} = \dot{Q}_{combustion} - \dot{Q}_{htg\ fuel}$$

$$\dot{Q}_{flue} = 1.7 \times 10^9 \frac{Btu}{hr} - 6.51 \times 10^7 \frac{Btu}{hr} = 1.635 \times 10^9 \frac{Btu}{hr}$$

Use $\dot{Q} = \dot{m} c_p \Delta T$ for the flue gas and calculate the exit temperature. Consider the entering flue gas as State 1 and the exiting flue gas as State 2.

$$\dot{Q}_{flue} = \dot{m}_{flue} c_p \Delta T = \dot{m}_{flue} c_p (T_2 - T_1)$$

$$T_2 = T_1 + \frac{\dot{Q}_{flue}}{\dot{m}_{flue} c_p}$$

$$T_2 = 1000^\circ F + \frac{1.635 \times 10^9 \frac{Btu}{hr}}{(2 \times 10^6 \frac{lb_m}{hr}) (0.25 \frac{Btu}{lb^\circ F})} = 4270^\circ F$$

Answer A